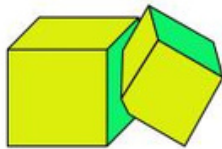


MK Team Level 11-12 Sample

Multiple-choice Questions

1. Two cubes of volumes V and W intersect. The part of the cube of volume V which is not common to the two cubes is 90% of its volume. The part of the cube of volume W which is not common to the two cubes is 85% of its volume. What is the relationship between V and W ?



(A) $V = W$

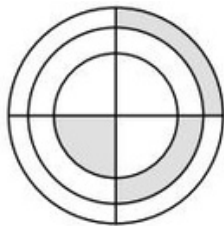
(B) $V = 3/2W$

(C) $V = 85/90W$

(D) $V = 90/85W$

(E) $V = 2/3W$

2. In the figure shown there are three concentric circles and two perpendicular diameters. If the three shaded figures have equal area and the radius of the small circle is one, what is the product of the three radii?



(A) $\sqrt{6}$

(B) 3

(C) $(3\sqrt{3})/2$

(D) $2\sqrt{2}$

(E) 6

3. The polynomial $5x^3 + ax^2 + bx + 24$ has integer coefficients a and b . Which of the following is certainly not a root of the polynomial?

(A) 1

(B) -1

(C) 3

(D) 5

(E) 6

4. Three kangaroos, Alex, Bob and Carl, go for a walk every day. If Alex doesn't wear a hat, then Bob wears a hat. If Bob doesn't wear a hat, then Carl wears a hat. Today Carl is not wearing a hat. Who is certainly wearing a hat today?

- (A) only Alex and Bob (B) only Alex
 (C) Alex, Bob and Carl (D) neither Alex nor Bob (E) only Bob

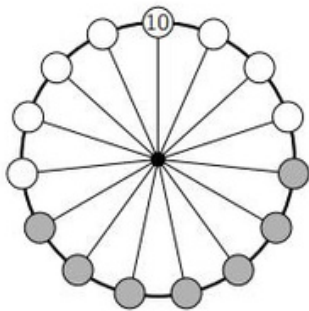
5. Diana cut up a rectangle of area 2016 into 56 equal squares. The lengths of the sides of the rectangle and of the squares are integers. For how many different rectangles is it possible for her to do this? (Two rectangles that can be obtained from one another by rotating 90 degrees are considered the same.)

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 0

6. Which of these five numbers does not divide $18^{2017} + 18^{2018}$?

- (A) 8 (B) 18 (C) 28 (D) 38 (E) 48

7. Matjaz has placed 15 numbers on a wheel. Only one of the numbers is visible, the 10 at the top. The sum of the numbers in any 7 consecutive positions on the wheel, such as the ones shaded grey, is always the same. When all 15 numbers are added, exactly how many of the numbers 75, 216, 365 and 2020 are possible totals?

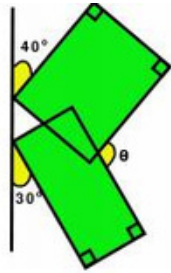


- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. Given five different positive integers a, b, c, d, e , we know that $c - e = b$, $a + b = d$ and $e - d = a$. Which of the numbers a, b, c, d, e is the largest?

- (A) a (B) b (C) c (D) d (E) e

9. Two rectangles are inclined to the vertical line at angles 40° and 30° as shown. What is the measure of the angle θ ?



- (A) 105° (B) 110° (C) 120° (D) 130° (E) 135°

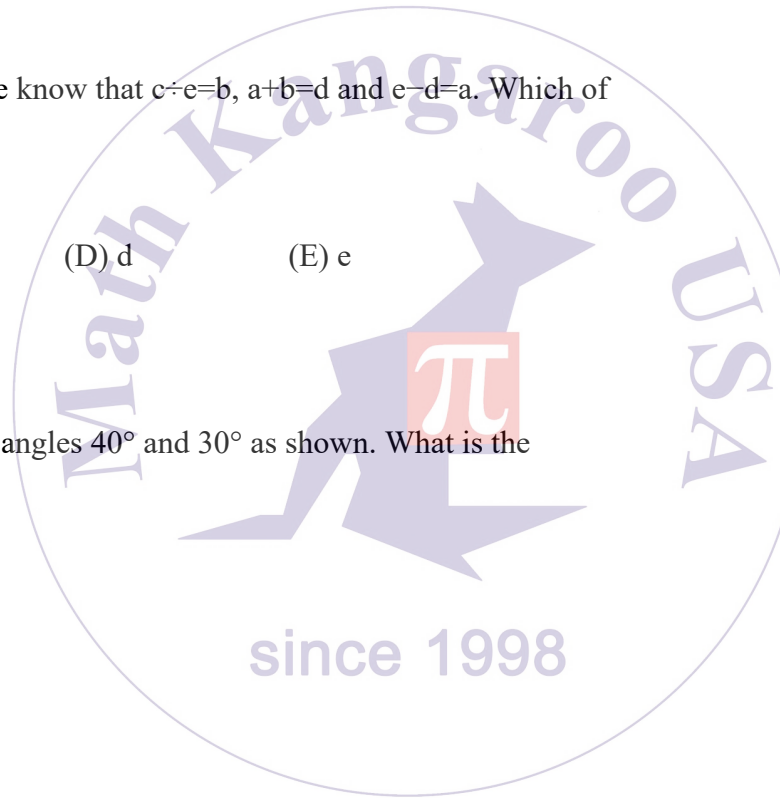
10. A box contains 4 chocolates and 1 fruit chew. John and Mary take turns drawing a treat out of the box without replacement. Whoever draws the fruit chew wins. John draws first. What is the probability that Mary wins?

- (A) $2/5$ (B) $3/5$ (C) $1/2$ (D) $5/6$ (E) $1/3$

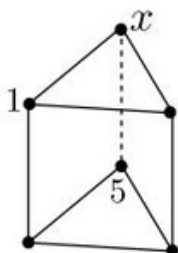
11. Determine the number of real solutions to the equation

$$(x^2 - 4x + 5)^{x^2 + x - 30} = 1.$$

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

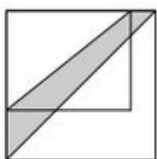


12. The prism in the figure is formed of two triangles and three squares. The six vertices are numbered from 1 to 6 in such a way that the sum of the four vertices of each square is the same for all three squares. Numbers 1 and 5 are already shown. What number is at the vertex labeled x ?



- (A) 2 (B) 3 (C) 4 (D) 6 (E) the situation is impossible

13. The length of one of the sides of a rectangular garden is increased by 20% and the length of the other side is increased by 50%. The new garden is a square, as shown in the diagram. The shaded area between the diagonal of the square garden and the diagonal of the original rectangular garden is 30 m^2 . What was the area of the original rectangular garden?



- (A) 60 m^2 (B) 65 m^2 (C) 70 m^2 (D) 75 m^2 (E) 80 m^2

14. Consider the sequence a_1, a_2, a_3, \dots with $a_1 = 2017$ and $a_{n+1} = (a_n - 1) / a_n$ for all $n \geq 1$. What is a_{2017} ?

- (A) -2017 (B) $-1/2016$ (C) $2016/2017$ (D) 1 (E) 2017

15. Let f be a function such that $f(x + y) = f(x)f(y)$ for all integers x and y . If $f(1) = 1/2$, find the value of $f(0) + f(1) + f(2) + f(3)$.

- (A) $1/8$ (B) $3/2$ (C) $5/2$ (D) $15/8$ (E) 6

16. In the morning, an ice-cream shop offers 16 flavors. Anna wants to choose a 2-flavor ice cream. In the evening, several flavors are sold out. Bella wants to choose a 3-flavor ice cream from the remaining flavors. Both Anna and Bella can choose from the same number of possible combinations. How many flavors were sold out?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

17. To calculate the result of $(a + b)/c$, Sara types $a + b \div c =$ on a calculator and the result is 11 (a , b , and c are positive integers). She then types $b + a \div c =$ and she is surprised to see that the result is 14. She realizes that the calculator is designed to calculate divisions before additions. What is the correct result of $(a + b)/c$?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

18. A quadratic function $f(x) = x^2 + px + q$ is such that its graph intersects the x -axis and the y -axis in three different points. The circle through these three points intersects the graph of f in a fourth point. What is the coordinate of this fourth point?

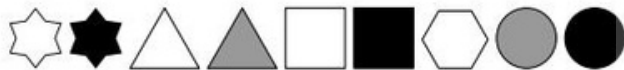
- (A) $(0, -q)$ (B) (p, q) (C) $(-p, q)$ (D) $(-\frac{q}{p}, \frac{q^2}{p^2})$ (E) $(1, p+q+1)$

19. Archimedes calculated $15!$. The result is written on the board. Unfortunately two of the figures, the second and the tenth, are not visible. Which are these two figures?

1 ■ 0 7 6 7 4 3 6 ■ 0 0 0

- (A) 2&0 (B) 4&8 (C) 7&4 (D) 9&2 (E) 3&8

20. Adam and Britt try to find out which of the following figures is Carl's favourite.



Adam knows that Carl has told Britt its shape. Britt knows that Carl has told Adam its colour. Then the following conversation takes place. Adam: "I don't know Carl's favourite figure and I know that Britt doesn't know it either." Britt: "At first I didn't know Carl's favourite figure, but now I do." Adam: "Now I know it too." Which figure is Carl's favourite?

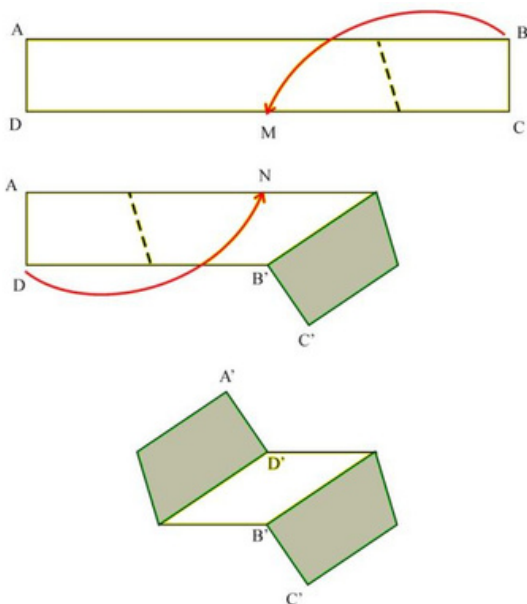


Fill-in-the-blank Questions

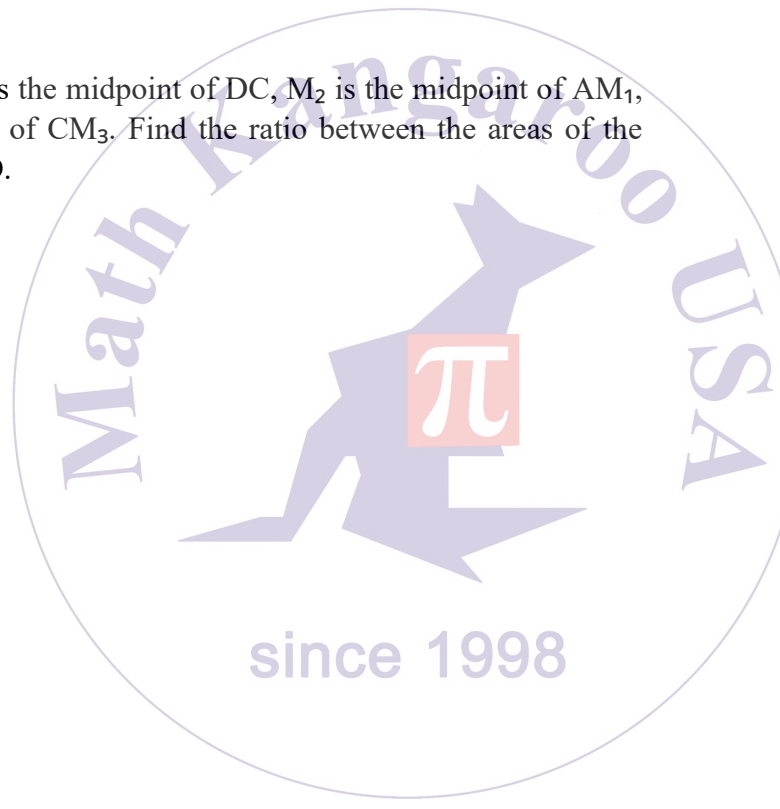
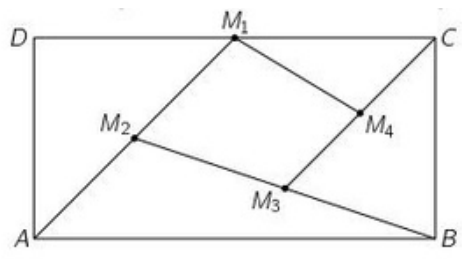
21. How many real solutions does the equation $|4^x - 3| - 2 = 1$ have?

22. An iceberg has the shape of a cube. Exactly 90% of its volume is hidden below the surface of the water. Three edges of the cube are partially visible over the water. The visible parts of these edges have lengths 24 m, 25 m and 27 m. How long is an edge of the cube?

23. A rectangular strip ABCD of paper 5 cm wide and 50 cm long is white on one side and grey on the other. Folding the strip, Cristina makes the vertex B coincide with the midpoint M of the side CD. Folding again, she makes the vertex D coincide with the midpoint N of the side AB. What is the area (in cm²) of the visible white part of the strip in the last picture?



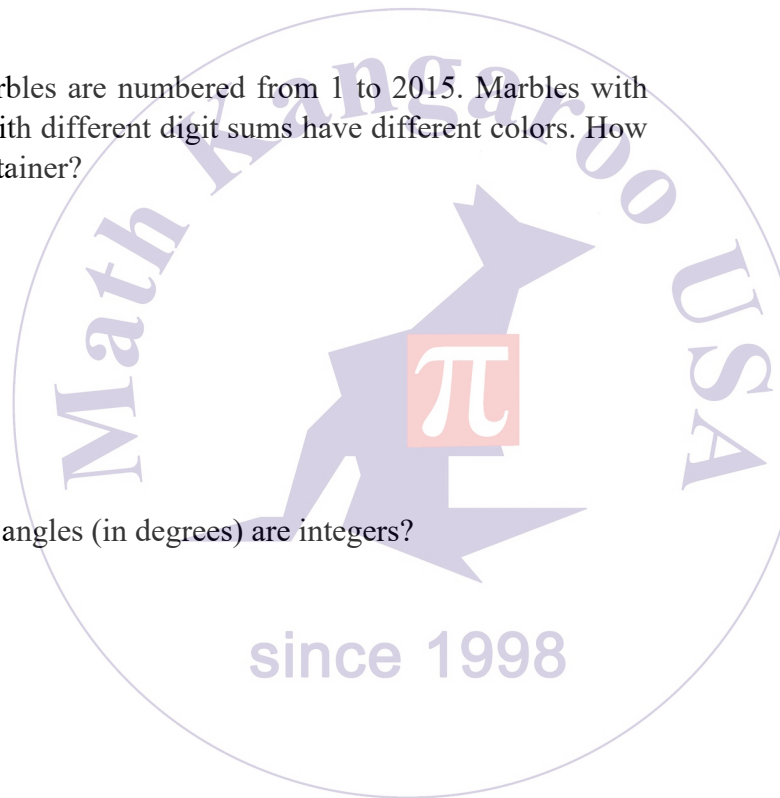
24. In the rectangle ABCD shown in the figure, M_1 is the midpoint of DC, M_2 is the midpoint of AM_1 , M_3 is the midpoint of BM_2 and M_4 is the midpoint of CM_3 . Find the ratio between the areas of the quadrilateral $M_1M_2M_3M_4$ and of the rectangle ABCD.



25. Three different numbers are chosen at random from the set $\{1,2,3,\dots,10\}$. What is the probability that one of them is the average of the other two?

26. The sum of five three-digit numbers is 2664, as shown below. What is the value of $A + B + C + D + E$?

A	B	C	
+	B	C	D
+	C	D	E
+	D	E	A
+	E	A	B
<hr/>			
2	6	6	4



27. There are 2015 marbles in a container. The marbles are numbered from 1 to 2015. Marbles with equal digit sums have the same color and marbles with different digit sums have different colors. How many different colors of marbles are there in the container?

28. How many regular polygons exist such that their angles (in degrees) are integers?

29. 96 members of a counting club are standing in a large circle. They start saying numbers 1, 2, 3, etc. in turn, going around the circle. Every member that says an even number steps out of the circle and the rest continue, starting the second round with 97. They continue in this way until only one member is left. Which number did this member say in the first round?

30. The square shown is filled with numbers in such a way that each row and each column contains the numbers 1, 2, 3, 4 and 5 exactly once. Moreover, the sum of the numbers in each of the three bold-bordered areas is equal. What number is in the upper right corner?

