Kangourou Sans Frontières

Math Kangaroo USA 2025

International Competition in Mathematics
Thursday, March 20, 2025

Math Kangaroo USA

Grades 11 and 12

You have 75 minutes to complete the test. ⋆ Calculators are not allowed. ⋆ Mark your answers on the answer sheet.

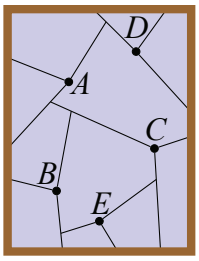
Problems 3 points each

1. The year 2025 is a perfect square because $2025 = 45^2$. How many years will pass until the next perfect square year?

(**A**) 25 (**B**) 91 (**C**) 121 (**D**) 500 (**E**) 2025

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2. A student threw five stones, one after the other, which hit a window at the points A, B, C, D, and E. When each stone hit the glass, it created one or more linear cracks that stopped either at a previous crack or at the edge of the window. In which order did he throw the stones?



(**A**) $DACBE$ (**B**) $ABCDE$ (**C**) $BDACE$
(**D**) $BCDAE$ (**E**) $DCABE$

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3. Vasily has 20 different colored balls: yellow, green, blue, and black. Of these, exactly 17 are not green, 15 are not black, and 12 are not yellow. How many balls are blue?

(**A**) 8 (**B**) 7 (**C**) 6 (**D**) 4 (**E**) 3

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4. In what interval does the value of 88×888 lie?

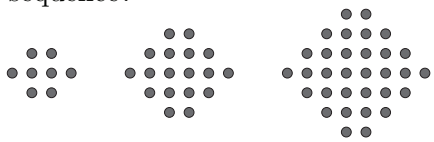
(**A**) between 8 and 88
(**B**) between 88 and 888
(**C**) between 888 and 8 888
(**D**) between 8 888 and 88 888
(**E**) between 88 888 and 888 888

5. Which of the following is equal to the square root of 16^{16}?

(**A**) 4^4 (**B**) 4^8 (**C**) 4^{16} (**D**) 8^8 (**E**) 16^4

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6. The shapes shown below are the first three shapes of a sequence. How many dots make up the fifth shape in the sequence?



(**A**) 72 (**B**) 74 (**C**) 76 (**D**) 78 (**E**) 80

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7. Mike obtains a number x by dividing the number $\sqrt{11}$ by 3. Where is the number x to be found on the number line?

(**A**) between 0 and 1 (**B**) between 1 and 2
(**C**) between 2 and 3 (**D**) between 3 and 4
(**E**) between 4 and 5

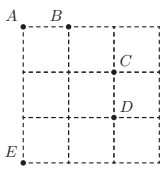
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8. Silia's favorite chocolate bars come in packets. Each packet used to contain five bars. Now they contain only four but are sold at the same price. By what percentage has the price of each bar increased?

(**A**) by 10% (**B**) by 20% (**C**) by 25%
(**D**) by 30% (**E**) by 50%

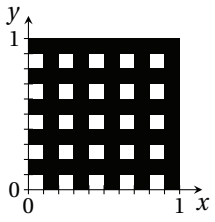
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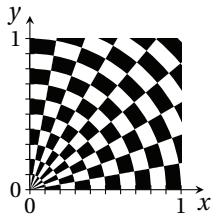
9. Robert wants to have four points so that the distances between each pair of points are different. Which of the points A, B, C, D, and E should he remove?

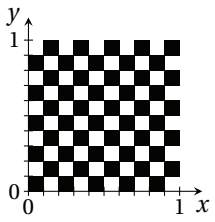


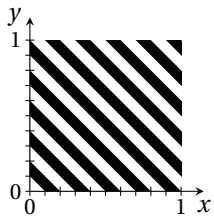
(**A**) A (**B**) B (**C**) C (**D**) D (**E**) E

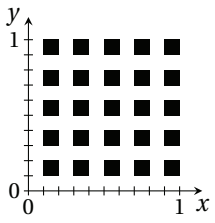
10. In the xy-plane, in the region defined by $0 \le x \le 1$, $0 \le y \le 1$, some points are painted black. The point (x, y) is painted black if for both x and y the first digit after the decimal point is odd. What does the result look like?

(**A**) 

(**B**) 

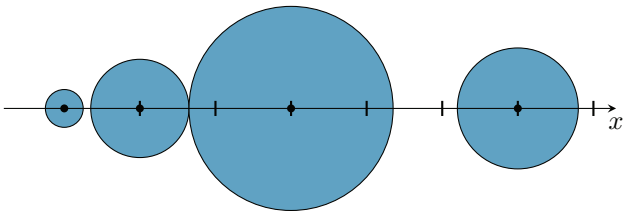
(**C**) 

(**D**) 

(**E**) 

Problems 4 points each

11. Four circular discs with positive radii r_1, r_2, r_3, and r_4 are centered at $(0, 0)$, $(1, 0)$, $(3, 0)$, and $(6, 0)$. The discs may touch but not overlap. What is the largest possible value of $r_1 + r_2 + r_3 + r_4$?



(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6

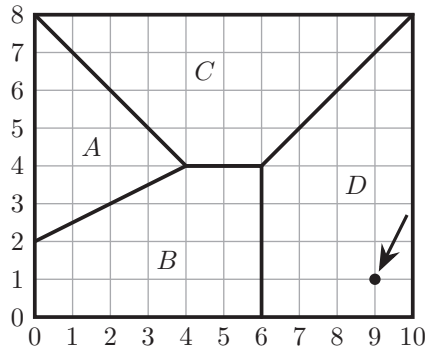
(**E**) There is no upper limit.

12. Among 10 different positive integers, there are exactly five that are divisible by 5 and exactly seven that are divisible by 7. Let M be the largest of these numbers. What is the smallest possible value of M?

(**A**) 105 (**B**) 77 (**C**) 75 (**D**) 63

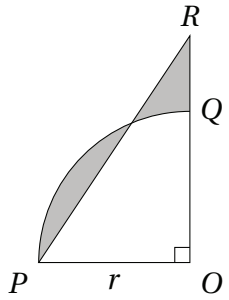
(**E**) another value

13. The map shows a small town which has 4 schools. It is split into four regions, A, B, C, and D, where each region contains one school and is defined as the set of all points that are closer to its respective school than to any other school. The coordinates of the school in region D are $(9, 1)$. What are the coordinates of the school in region A?



(**A**) (0,4) (**B**) (1,4) (**C**) (1,5) (**D**) (1,6) (**E**) (2,4)

14. The diagram shows a quarter-circle OPQ and a triangle OPR, where the length of OP is r. The two shaded regions have the same area. What is the length of OR?



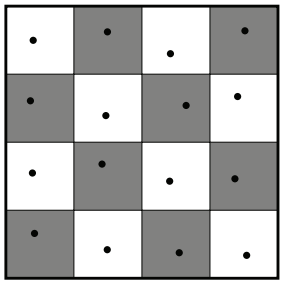
(**A**) $\frac{\pi r}{2}$ (**B**) $\frac{3r}{2}$ (**C**) πr (**D**) $\frac{2}{\pi}$ (**E**) $\frac{\pi}{2r}$

15. What is the smallest positive integer N such that $\sqrt{2\sqrt{3\sqrt{N}}}$ is an integer?

(**A**) $2^{12} \cdot 3^6$ (**B**) $2^4 \cdot 3^{14}$ (**C**) $2^4 \cdot 3^6 \cdot 5^8$

(**D**) $2^4 \cdot 3^2$ (**E**) none of the previous

16. On a giant 4×4 chessboard there are 16 kangaroos, one in each square. On each turn, each of the kangaroos jumps to a neighboring square (up, down, left, or right, but not diagonally). All kangaroos stay on the board. There can be multiple kangaroos on any square. After 100 turns, what is the largest possible number of empty squares?



(**A**) 15 (**B**) 14 (**C**) 12 (**D**) 10 (**E**) 8

17. The five digit number $\overline{N18NN}$ is divisible by 18. Which of the following statements is true about the digit N?

(**A**) There is exactly one such N.

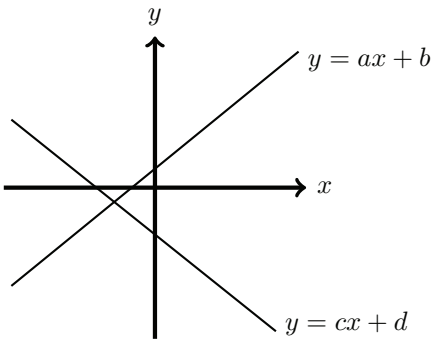
(**B**) There are exactly two such N.

(**C**) There are exactly three such N.

(**D**) No such N exists.

(**E**) There are more than three such N.

18. A student drew the graphs of two linear functions in a coordinate system, as shown.



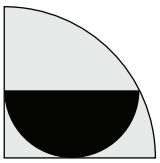
The expression $ab + cd - (ac + bd)$ is always

(**A**) negative. (**B**) non-positive.

(**C**) positive. (**D**) zero.

(**E**) None of the above is always true.

19. The area of the black semicircle is 12 cm^2. What is the area of the big quarter circle?



(**A**) 42 cm^2 (**B**) 36 cm^2 (**C**) 32 cm^2

(**D**) 30 cm^2 (**E**) 25 cm^2

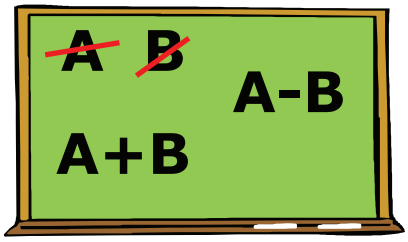
20. When grandma started knitting woolen socks, she had a huge ball of yarn with a diameter of 30 cm. After finishing 70 socks, she still has a ball of yarn with a diameter of 15 cm. How many more socks can grandma knit with the yarn that is left?



(**A**) 70 (**B**) 50 (**C**) 30 (**D**) 20 (**E**) 10

Problems 5 points each

21. A student starts with two numbers written on the board. He then deletes them and writes the sum of the numbers and the positive difference of the numbers. He continues the same process with the new numbers. He starts with the numbers 3 and 5 and repeats the process 50 times. What are the two numbers he will end up with?



(**A**) 3^{25} and 5^{25} (**B**) 3^{50} and 5^{50}

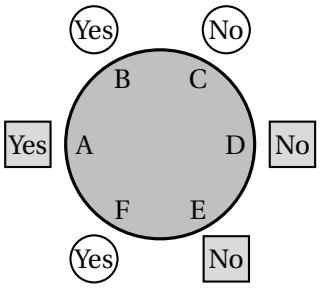
(**C**) $2 \cdot 3^{25}$ and $2 \cdot 5^{25}$ (**D**) $3 \cdot 2^{25}$ and $5 \cdot 2^{25}$

(**E**) none of the above

22. John wrote an arbitrary two-digit integer on the blackboard. Then, he erased the last digit of the integer. As a result, the initial integer decreased by $p\%$. Which of the following is the closest to the largest possible value of p?

(**A**) 10 (**B**) 50 (**C**) 90 (**D**) 95 (**E**) 99

23. A group of three square men from Mars and a group of three circular men from Jupiter are sitting around a table, as shown. One of the six has the key to their flying saucer. All members of one group always tell the truth and all members of the other group always lie. All six were asked the question, "Does a person sitting next to you have the key?" Their answers are shown in the figure. Who has the key?



(**A**) A (**B**) B (**C**) C (**D**) D (**E**) E

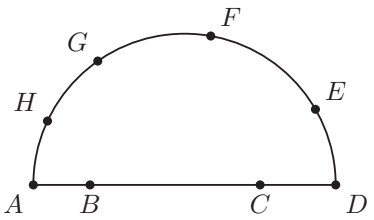
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24. Julia and her little sister Paula go out together for a bike ride. Both ride at a constant speed, Julia at 18 km/h and Paula at 12 km/h, and they follow the same path. Julia feels tired after 20 minutes and decides to go back. When she meets Paula, Julia tells her to turn around and both return home, each at her own speed. How many minutes later than Julia will Paula arrive home?

(**A**) 4 (**B**) 6 (**C**) 8 (**D**) 10 (**E**) 15

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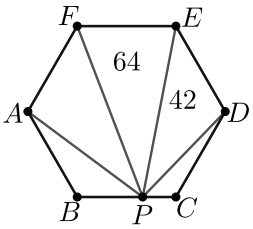
25. On a semicircle with diameter AD, points B and C lie on the diameter, and points E, F, G, and H lie on the arc of the semicircle. How many triangles can be formed with their vertices at three of these eight points?



(**A**) 15 (**B**) 50 (**C**) 51 (**D**) 52 (**E**) 54

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26. The diagram shows a regular hexagon $ABCDEF$. Point P lies on BC so that the area of $\triangle PEF$ is 64 and the area of $\triangle PDE$ is 42. What is the area of $\triangle APF$?



(**A**) 53 (**B**) 54 (**C**) 56 (**D**) 60 (**E**) 64

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27. Three boxes contain three balls each. The inscriptions on the lids show the contents of each box. The lids are rearranged so that none of them shows the contents correctly.

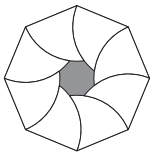


Mike picks a box, randomly removes a ball from it, and notes its color without putting it back. What is the smallest number of balls that Mike needs to remove to determine the contents of each box?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

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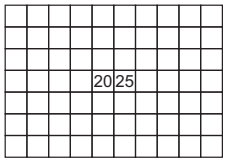
28. The figure shows a regular octagon of side 1 cm. An arc of radius 1 cm has been drawn centered at each vertex as shown. What is the perimeter of the shaded region?



(**A**) π cm (**B**) $\frac{2\pi}{3}$ cm (**C**) $\frac{8\pi}{9}$ cm (**D**) $\frac{4\pi}{5}$ cm (**E**) $\frac{3\pi}{4}$ cm

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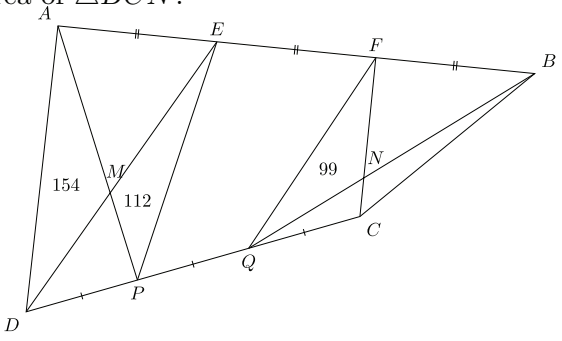
29. Patricia has written a number in each cell of a 7×10 table. The sum of all the numbers in any 3×4 or 4×3 rectangle is zero. The numbers in two of the cells are shown in the diagram. What is the sum of all the numbers in the table?



(**A**) -5 (**B**) -20 (**C**) -25 (**D**) -45 (**E**) It is not possible to determine.

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30. The sides AB and CD of the convex quadrilateral $ABCD$ are each divided into three parts by points E, F, P, and Q so that $AE = EF = FB$ and $DP = PQ = QC$. The diagonals of $AEPD$ and $FBCQ$ intersect at M and N respectively. The areas of $\triangle AMD$, $\triangle EMP$, and $\triangle FNQ$ are 154, 112, and 99, respectively. What is the area of $\triangle BCN$?



(**A**) 57 (**B**) 70 (**C**) 72 (**D**) 86 (**E**) 141